Section 9-2, Mathematics 104

Product and Quotient Rules for Radicals

Were going to look at two rules that will help simplify radical expressions

$$\sqrt[n]{u\cdot v}=\left(u\cdot v\right)^{1/n}=u^{1/n}\cdot v^{1/n}=\sqrt[n]{u}\cdot\sqrt[n]{v}$$

$$\sqrt[n]{\frac{u}{v}}=\left(\frac{u}{v}\right)^{1/n}=\frac{u^{1/n}}{v^{1/n}}=\frac{\sqrt[n]{u}}{\sqrt[n]{v}}$$

Let's see how these rules can be useful

Examples:

$$\sqrt{75}=\sqrt{25}\cdot\sqrt{3}=5\sqrt{3}$$

$$\frac{\sqrt{162}}{\sqrt{10}}=\frac{\sqrt{81}\cdot\sqrt{2}}{\sqrt{5}\cdot\sqrt{2}}=\frac{9}{\sqrt{5}}$$

$$\sqrt{25x^2}=\sqrt{25}\cdot\sqrt{x^2}=5\left|x\right|$$

$$\sqrt{72x^3y^2}=\sqrt{36}\cdot\sqrt{2}\cdot\sqrt{x^3}\cdot\sqrt{y^2}=6\sqrt{2}x\sqrt{x}\left|y\right|=6x\left|y\right|\sqrt{2x}$$

$$\sqrt[3]{40}=\sqrt[3]{2\cdot2\cdot2\cdot5}=\sqrt[3]{2^3}\cdot\sqrt[3]{5}=2\sqrt[3]{5}$$

$$\frac{\sqrt{56x^2}}{\sqrt{8}}=\frac{\sqrt{2\cdot2}\cdot\sqrt{3\cdot3}\cdot\sqrt{x^2}}{\sqrt{2}\cdot\sqrt{4}}=\frac{\cancel{\sqrt{2}}\cdot\sqrt{2}\cdot3\left|x\right|}{\cancel{\sqrt{2}}\cdot\sqrt{4}}=\frac{3\sqrt{2}\left|x\right|}{2}$$

$$\sqrt[3]{\frac{y^5}{27x^3}}=\frac{\sqrt[3]{y^5}}{\sqrt[3]{27x^3}}=\frac{\sqrt[3]{y^3}\cdot\sqrt[3]{y^2}}{\sqrt[3]{27}\cdot\sqrt[3]{x^3}}=\frac{y\sqrt[3]{y^2}}{3x}$$

Rationalizing Denominators

Rationalizing numeric denominators is an esthetic that I don't agree with.
That is, I don't think that

$\frac{\sqrt{2}}{2}$ is simpler than $\frac{1}{\sqrt{2}}$

However rationalizing a radical with a variable in the denominator is important.

To rationalize

$\frac{1}{\sqrt{3}}$ we find a value that will remove the radical from the denominator.

Examples:

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\frac{4}{\sqrt[3]{9}} = \frac{4}{\sqrt[3]{3 \cdot 3}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{4\sqrt[3]{3}}{\sqrt[3]{3^3}} = \frac{4\sqrt[3]{3}}{3}$$

Adding and Subtracting Radical Expressions

When adding expressions with radicals, only like radicals can be combined.

Examples:

$$\sqrt{7}+5\sqrt{7}-2\sqrt{7}=\sqrt{7}\left(1+5-2\right)=4\sqrt{7}$$

$$3\sqrt[3]{x}+2\sqrt[3]{x}+\sqrt{x}-8\sqrt{x}=\sqrt[3]{x}\left(3+2\right)+\sqrt{x}\left(1-8\right)=5\sqrt[3]{x}-7\sqrt{x}$$

Simplifying first

$$\sqrt{45x}+3\sqrt{20x}=\sqrt{9\cdot 5x}+3\sqrt{4\cdot 5x}=3\sqrt{5x}+6\sqrt{5x}=9\sqrt{5x}$$

$$6\sqrt{\frac{24}{x^4}}-3\sqrt{\frac{54}{x^4}}=6\sqrt{\frac{4\cdot 6}{x^4}}-3\sqrt{\frac{9\cdot 6}{x^4}}=12\sqrt{\frac{6}{x^4}}-9\sqrt{\frac{6}{x^4}}=3\sqrt{\frac{6}{x^4}}=$$
$$3\frac{\sqrt{6}}{\sqrt{x^2}\cdot\sqrt{x^2}}=\frac{3\sqrt{6}}{x^2}$$

Rationalizing first

$$\sqrt{7}-\frac{5}{\sqrt{7}}=\sqrt{7}-\frac{5}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}=\sqrt{7}-\frac{5\sqrt{7}}{7}=\sqrt{7}\left(1-\frac{5}{7}\right)=\frac{2\sqrt{7}}{7}$$

Multiplying and Dividing Radical Expressions

Examples:

$$\sqrt{6}\cdot\sqrt{3}=\sqrt{2\cdot3\cdot3}=3\sqrt{2}$$

$$\sqrt[3]{5}\cdot\sqrt[3]{16}=\sqrt[3]{5\cdot2\cdot2^3}=2\sqrt[3]{10}$$

$$\sqrt{3}\left(2+\sqrt{5}\right)=2\sqrt{3}+\sqrt{3\cdot5}=2\sqrt{3}+\sqrt{15}$$

Using FOIL

$$\left(2\sqrt{7}-4\right)\left(\sqrt{7}+1\right)=2\sqrt{7}\sqrt{7}+2\sqrt{7}-4\sqrt{7}-4=14-2\sqrt{7}-4=10-2\sqrt{7}$$

Conjugates

The idea of a radical conjugate takes advantage of the identity

$$A^2-B^2=\left(A-B\right)\left(A+B\right)$$

If A or B is a square root radical, multiplying by the conjugate will remove it.

Example:

$$\left(1-\sqrt{3}\right)$$

The conjugate is

$$\left(1+\sqrt{3}\right)$$

So

$$\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)=1^2-\left(\sqrt{3}\right)^2=1-3=-2$$

Using conjugates to rationalize a denominator

Examples:

$$\frac{\sqrt{3}}{1-\sqrt{5}}\cdot\frac{1+\sqrt{5}}{1+\sqrt{5}}=\frac{\sqrt{3}+\sqrt{15}}{1-5}=-\frac{\sqrt{3}+\sqrt{15}}{4}=$$

$$\frac{1}{\sqrt{x}-\sqrt{x+1}}\cdot\frac{\sqrt{x}+\sqrt{x+1}}{\sqrt{x}+\sqrt{x+1}}=\frac{\sqrt{x}+\sqrt{x+1}}{x-(x+1)}=-\left(\sqrt{x}+\sqrt{x+1}\right)$$